Computational Challenges in Self-Consistent Halo Modeling and Beam-Beam Simulation

Ji Qiang
Lawrence Berkeley National Laboratory

Halo'03 – ICFA Advanced Beam Dynamics Workshop
May 19 – 23, Montauk, NY

Work performed under the auspices of the
DOE Advanced Computing for 21st Century Accelerator Science and Technology Project using resources at the
Center for Computational Sciences and the
National Energy Research Scientific Computing Center

Computational Models

- Halo Modeling: Poisson-Vlasov Equation

$$L{:}f(\mathbf{r},\mathbf{p}) = 0$$

$$\nabla^2\phi \quad = \ - \iiint f(r,p)d^3p$$

- Beam-Beam Simulation: Poisson-Vlasov Equation

$$L{:}f_1(\mathbf{r},\mathbf{p}) = 0 \quad L{:}f_2(\mathbf{r},\mathbf{p}) = 0$$

$$\nabla^2\phi_1 \quad = \ - \iiint f_2(r,p)d^3p$$

$$\nabla^2\phi_2 \quad = \ - \iiint f_1(r,p)d^3p$$

Computational Methods

- Particle-based methods:
  Approximate the distribution function using macroparticles and follow the dynamics of individual particles
- Direct solver:
  Find the time evolution of the distribution function directly

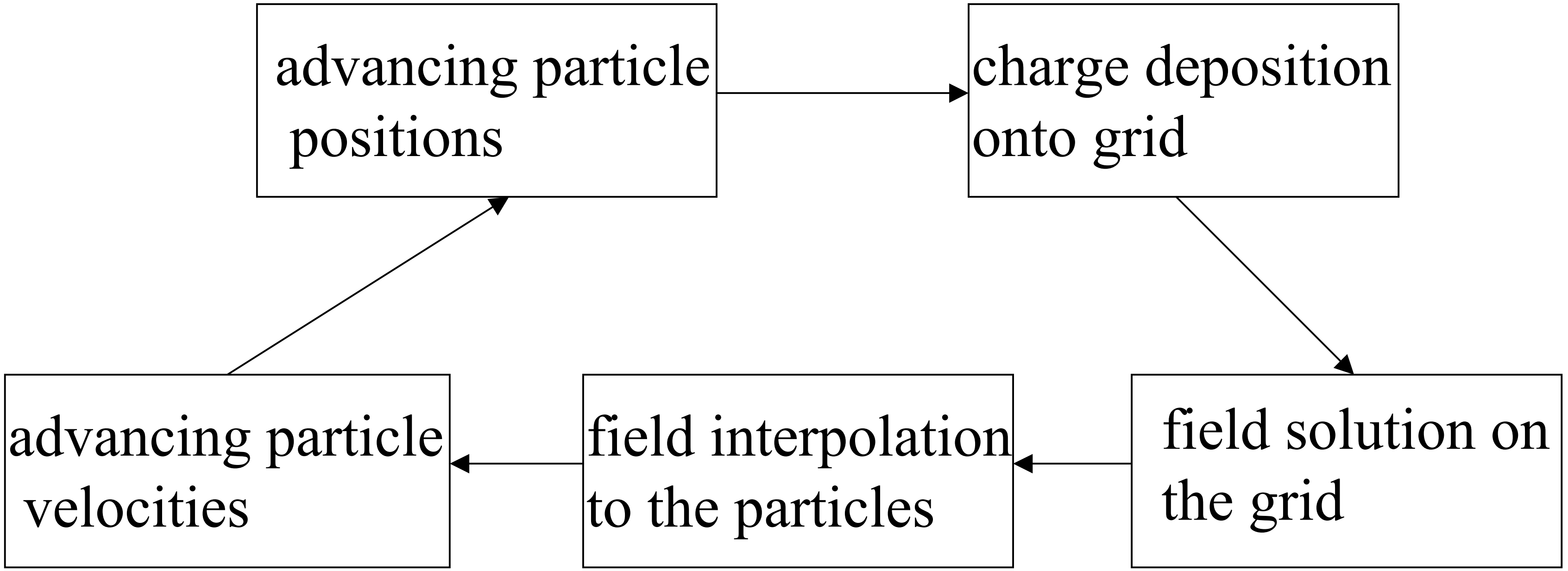
Particle-Based Method: Pros and Cons

- Pros
  1. Easy to implement
  2. Flexible to handle multiple physics, e.g. radiation, particle-wall interaction, …..
- Cons:
  1. Statistical noise
  2. Numerical collisionality

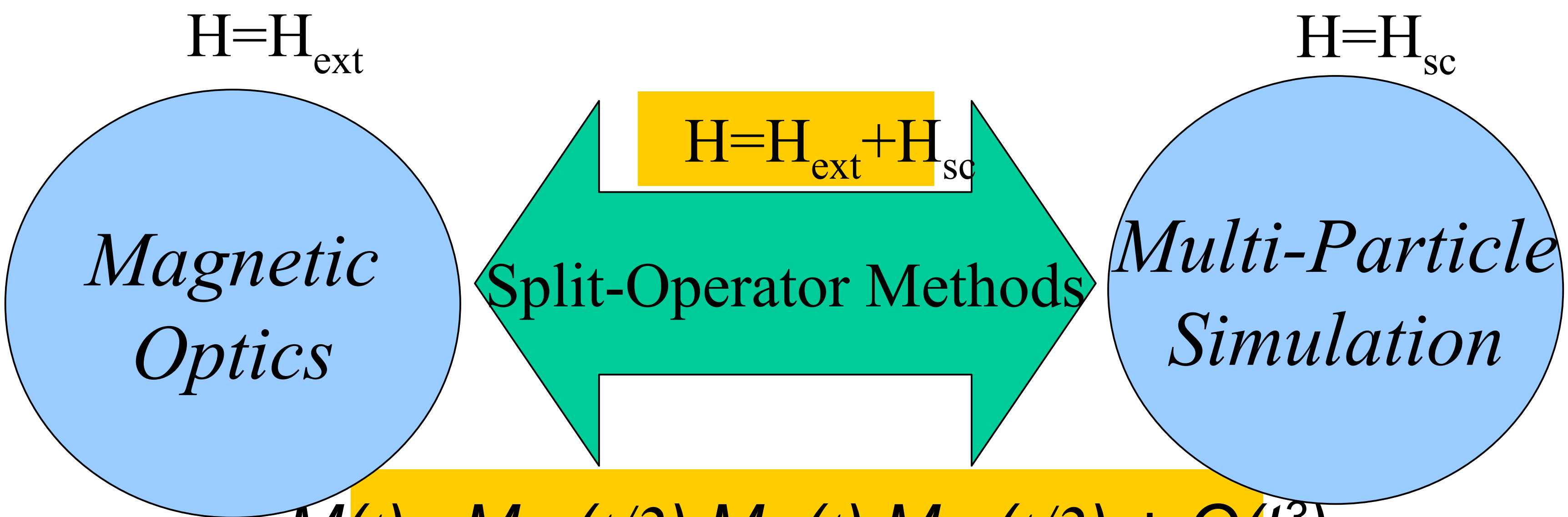
Direct Solver: Pros and Cons

- Pros:
  1. No numerical noise
- Cons:
  1. Subject to numerical instability, negative distribution function
  2. Not easy to be extended to higher dimension, e.g. 6D

Particle-In-Cell Method



Split-Operator Approach



$$M(t)= M_{ext}(t/2)\, M_{sc}(t)\, M_{ext}(t/2) + O(t^3)$$

$M=M_{ext}$ $M=M_{sc}$

(arbitrary order possible via Yoshida)

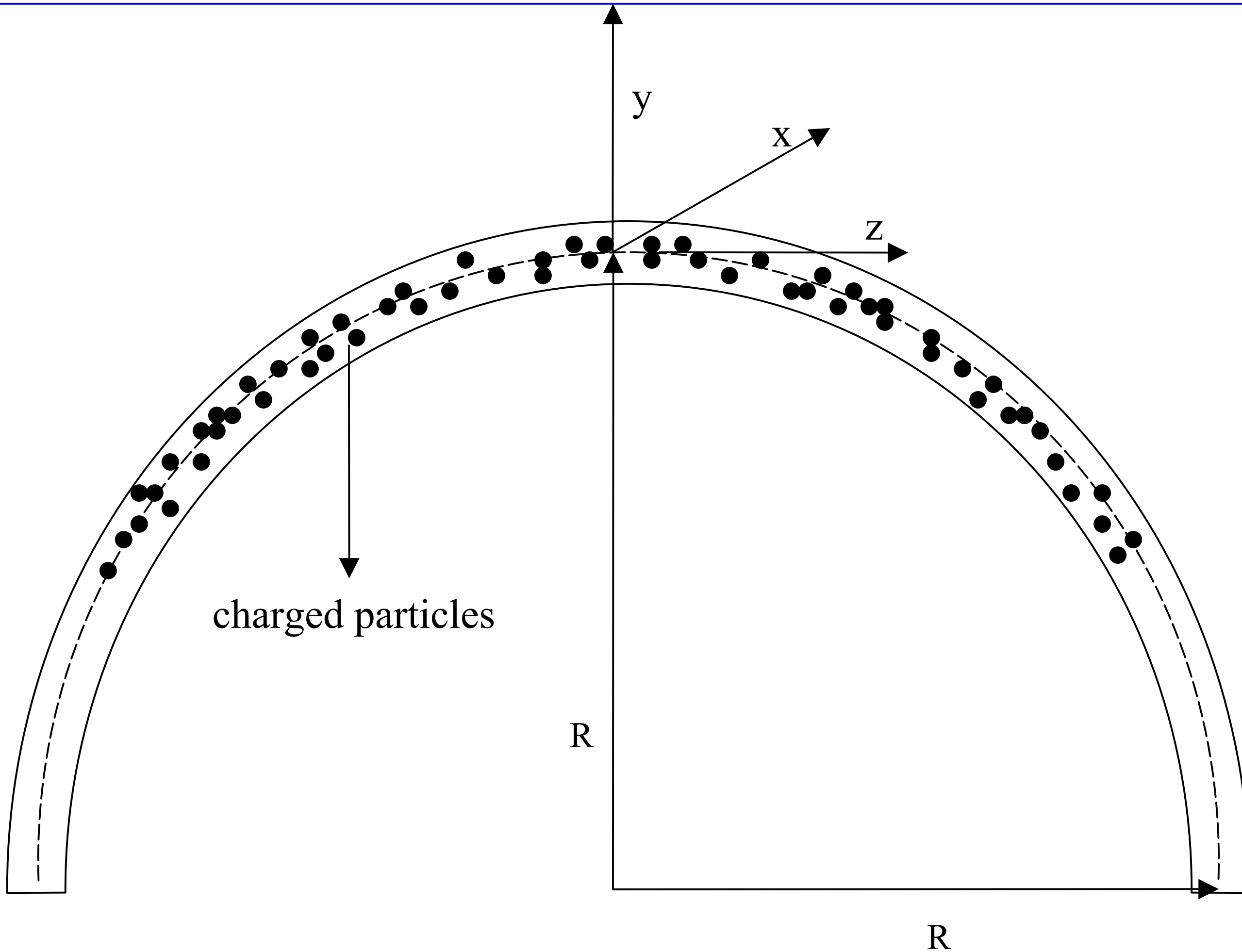
Direct Solver

- Spectral method
- Finite difference method

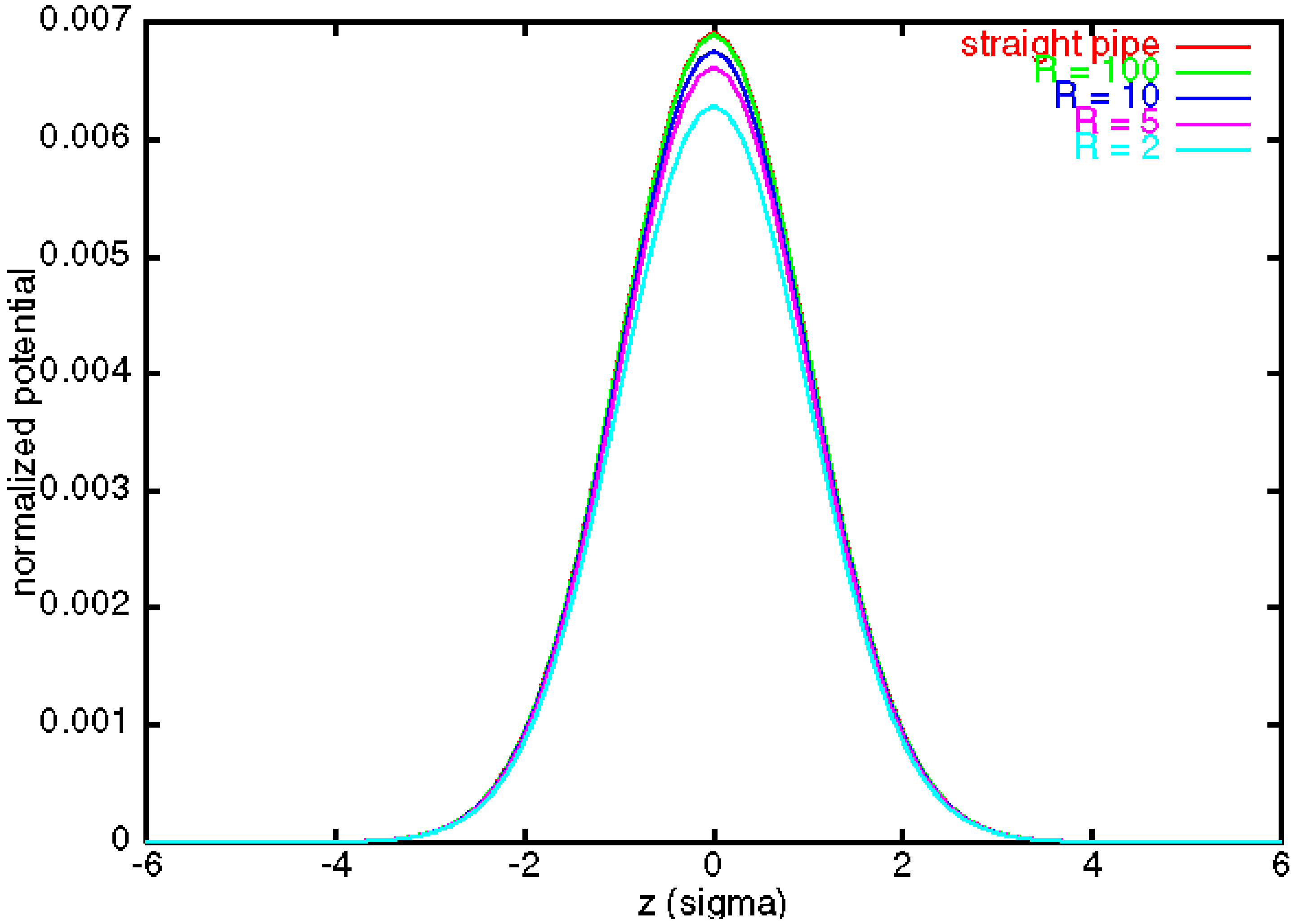
Numerical Solver of the Poisson Equation

- Simple geometry and boundary conditions:
  - Spectral method: Fourier base, Bessel function, …
- Complex geometry:
  - Finite element method
  - Finite difference method
- Efficiency!!!!

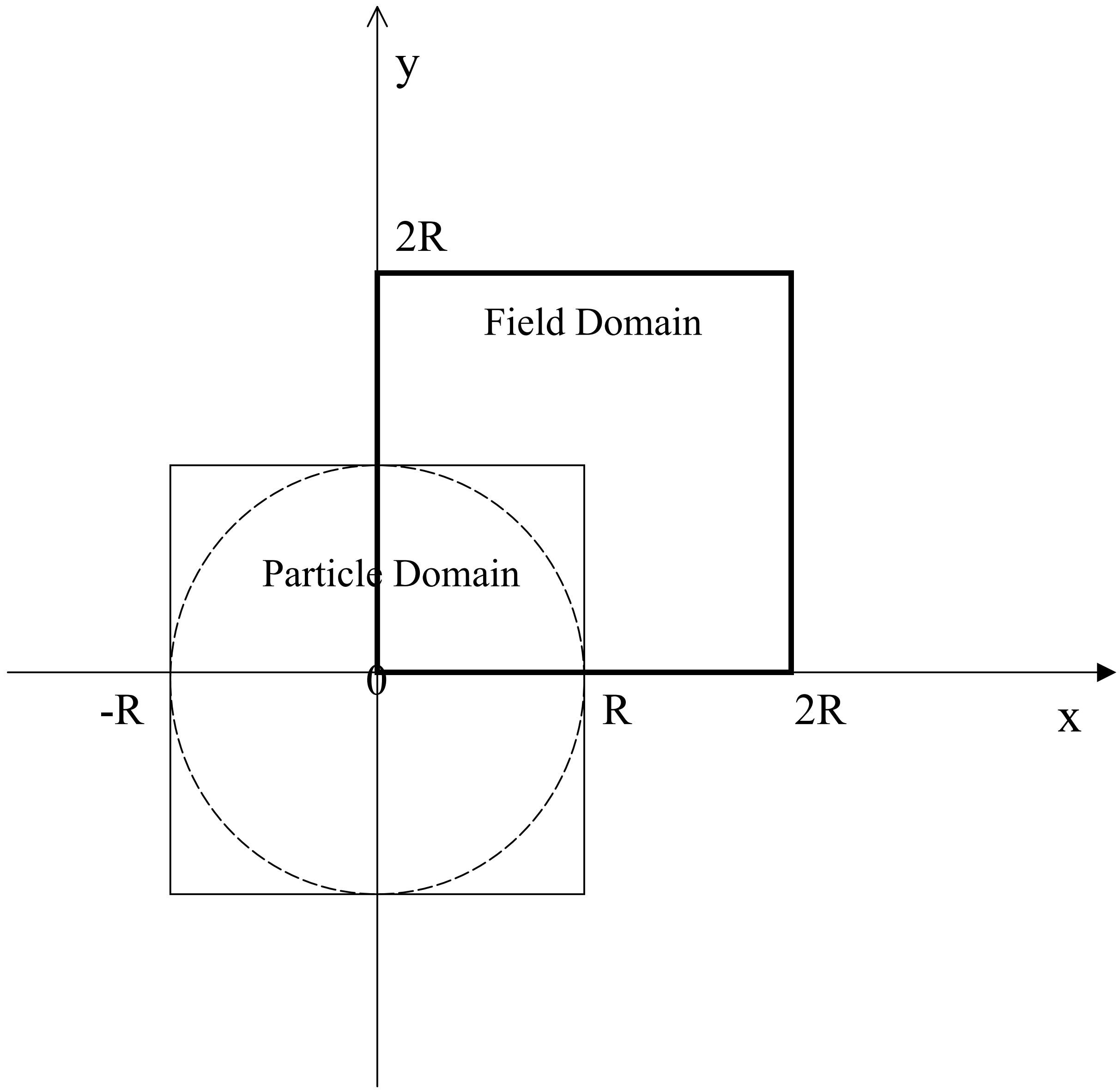
Example 1: Long Bunch with Large Aspect Ratio (cont'd)



Example 1: Electric Potential vs. Z with Aspect 100 (spectral method with Hermite-Gaussian approximation)

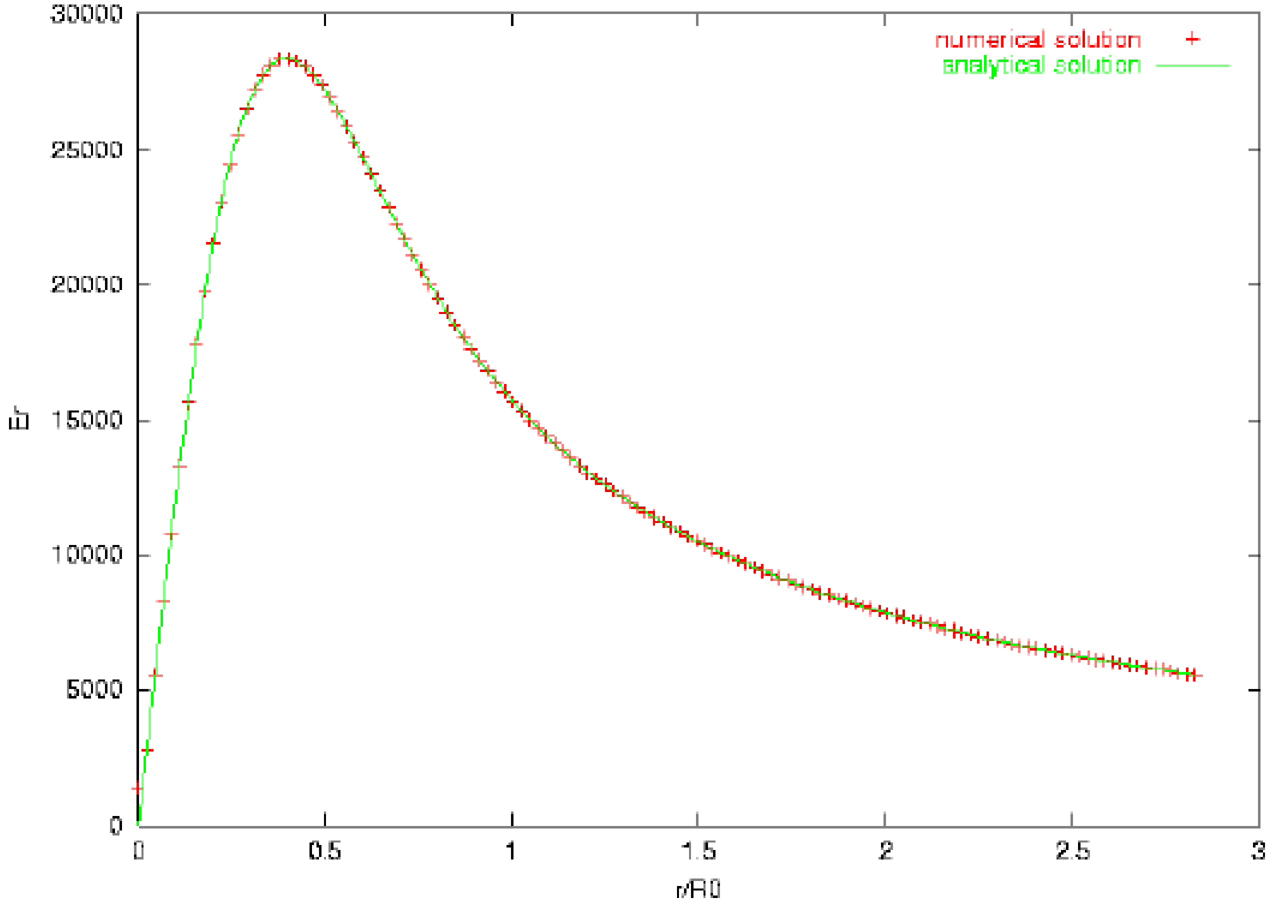


Example 2: Long Range Beam-Beam Force (cont'd)



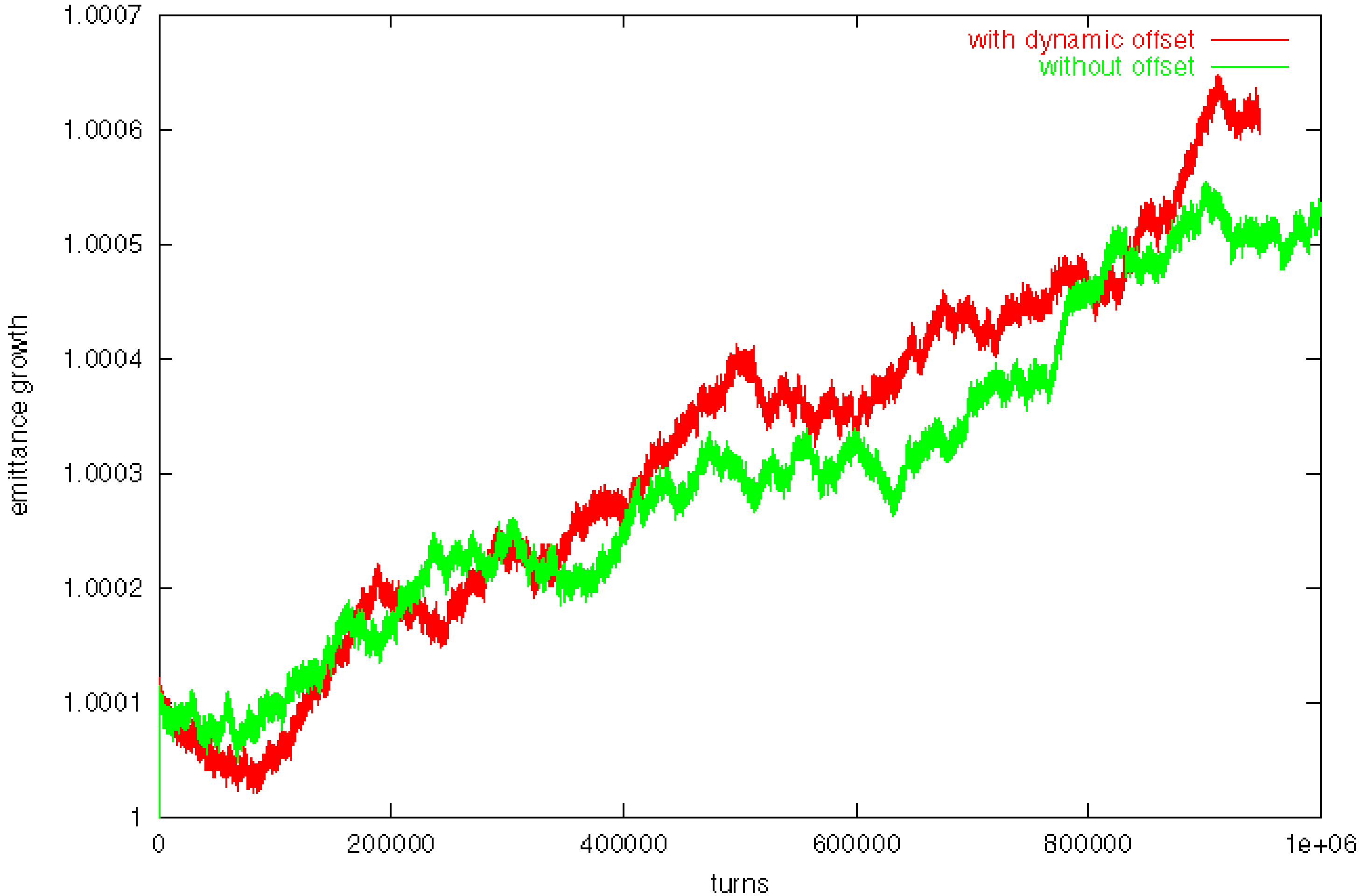
Example 2: Comparison between Numerical Solution and Analytical Solution (shifted-Green function method)

Radial Electric Field vs. Distance inside the Field Domain with Gaussian Density Distribution



Example 3: Long Time Small Emittance Growth: Physical or Numerical?

(strong-strong beam-beam in LHC)



Computational Challenges:

- Efficient Poisson solver on parallel computer
- Large number of particles
- Long time tracking
- Stable direct solver
- Other physical effects:
  - Multiple bunch coupling
  - Electron cloud
  - Intrabeam scattering
  - Gas scattering
  - Large angle collision
  - Wakefields

Can We Meet These Challenges? $$$$$$……